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DTP/00/18

New Insights into the Perturbative Structure of Electroweak Sudakov Logarithms: Breakdown of Conventional Exponentiation

W. Beenakker[*†] **and A. Werthenbach**[‡§]

Department of Physics, University of Durham, Durham DH1 3LE, U.K.

Abstract

To match the expected experimental precision at future linear colliders, improved theoretical predictions beyond next-to-leading order are required. At the anticipated energy scale of $\sqrt{s} = 1$ TeV the electroweak virtual corrections are strongly enhanced by collinear-soft Sudakov logarithms of the form $\log^2(s/M^2)$, with M being the generic mass scale of the W and Z bosons. By choosing an appropriate gauge, we have developed a formalism to calculate such corrections for arbitrary electroweak processes. As an example we consider in this letter the process $e^+e^- \to f\bar{f}$. In unbroken theories like QED and QCD the Sudakov form factor, resummed to all orders in perturbation theory, simply amounts to exponentiation of the one-loop corrections. However, based on an explicit two-loop calculation we find non-exponentiating terms, originating from the mass gap between the photon and the Z boson in the neutral sector of the Standard Model.

[*]W.J.P.Beenakker@durham.ac.uk

[†]Supported by a PPARC Research Fellowship

[‡]Anja.Werthenbach@durham.ac.uk

[§]Supported by a DAAD Doktorandenstipendium (HSP III)

1 Introduction

At the next generation of colliders center-of-mass energies will be reached that largely exceed the electroweak scale. For instance, the energy at a future linear e^+e^- collider is expected to be in the TeV range [1]. At these energies one enters the realm of large perturbative corrections. Even the effects arising from weak corrections are expected to be of the order of 10% or more [2, 3], i.e. just as large as the well-known electromagnetic corrections. In order not to jeopardize any of the high-precision studies at these high-energy colliders, it is therefore indispensable to improve the theoretical understanding of the radiative corrections in the weak sector of the Standard Model (SM). In particular this will involve a careful analysis of effects beyond first order in the perturbative expansion in the (electromagnetic) coupling $\alpha = e^2/(4\pi)$.

The dominant source of radiative corrections at TeV-scale energies is given by logarithmically enhanced effects of the form $\alpha^n \log^m(M^2/s)$ for $m \leq 2n$, involving particle masses M well below the collider energy $\sqrt{s}$. A natural way of controlling the theoretical uncertainties would therefore consist in a comprehensive study of these large logarithms, taking into account all possible sources (i.e. ultraviolet, soft, and collinear). In first approximation the so-called Sudakov logarithms $\propto \alpha^n \log^{2n}(M^2/s)$, arising from collinear-soft singularities [4], constitute the leading contribution to the large electroweak correction factors. Recent studies have focused on these Sudakov effects in the process $e^+e^- \to f\bar{f}$ [5]–[7]. Unfortunately the three independent studies are in mutual disagreement, exhibiting strikingly different higher-order results already for the virtual corrections. The main cause for the differences can be traced back to the use of different assumptions concerning the exponentiation properties of the Sudakov logarithms in the SM. Many of these assumptions are based on the analogy with unbroken theories like QED and QCD, where the resummation of the higher-order effects amounts to an exponentiation of the one-loop corrections (see for instance Refs. [4] and [8]–[10]). However, the SM is a broken gauge theory with a large mass gap in the neutral sector between the massless photon and the massive Z boson. As such it remains to be seen how much of the analogy with unbroken theories actually pertains to the SM.

In this letter we therefore focus on the virtual Sudakov logarithms in the reaction $e^+e^- \to f\bar{f}$, in an attempt to clarify how the $\mathcal{O}(\alpha^2)$ effects relate to the $\mathcal{O}(\alpha)$ ones. In this way we identify to what extent the SM behaves like an unbroken theory at high energies, which is a piece of information that may proof invaluable for performing an all-order resummation of Sudakov logarithms into a Sudakov form factor. Moreover, since the Sudakov logarithms originate from the exchange of soft, effectively on-shell gauge bosons, many of the features derived for the virtual corrections are intimately related to properties of the corresponding real-gauge-boson emission processes.

2 Electroweak Sudakov logarithms in $e^+e^- \to f\bar{f}$

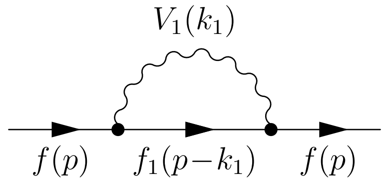
In order to facilitate the calculation of the one- and two-loop Sudakov logarithms, we work in the Coulomb gauge for both massless and massive gauge bosons (the subtleties associated with the Coulomb gauge for massive gauge bosons will be discussed in Ref. [11]). Working in this special gauge has the advantage that all virtual Sudakov logarithms are contained exclusively in the self-energies of the external on-shell particles [9, 11] or the self-energies of any intermediate particle that happens to be effectively on-shell.[1] The latter is, for instance, needed for the production of near-resonance unstable particles. The elegance of this method lies in its universal nature, with the Sudakov logarithms originating from vertex, box etc. corrections being suppressed. Once all self-energies to all on-shell/on-resonance SM particles have been calculated, the prediction of the Sudakov form factor for an *arbitrary* electroweak process becomes trivial. The relevant self-energies for the calculation of the Sudakov logarithms involve the exchange of collinear-soft gauge bosons. The collinear-soft exchange of fermions, scalars and ghosts leads to suppressed contributions, since the propagators of these particles do not have the required pole structure.

2.1 The fermionic self-energy at one-loop level

As mentioned above, in order to determine the Sudakov logarithms in the process $e^+e^- \to f\bar{f}$ one has to calculate the external self-energies (i.e. the wave-function factors) of all fermions involved in the process.

Consider to this end the fermionic one-loop self-energy $\Sigma^{(1)}(p, n, M_1)$, originating from the emission of a gauge boson V_1 with loop-momentum k_1 and mass M_1 from a fermion f with momentum p:

$$-\,i\,\Sigma^{(1)}(p, n, M_1) \quad =$$



Here n is the unit vector in the time direction, which enters by virtue of using the Coulomb gauge. In the high-energy limit the fermion mass in the numerator of the fermion propagator can be neglected. The self-energy $\Sigma^{(1)}$ then contains an odd number of γ-matrices, leading to the following natural decomposition in terms of the two possible structures $\not{p}$ and $\not{n}$:

$$\Sigma^{(1)}(p, n, M_1) \quad \approx \quad \Big[\not{p}\,\Sigma_p^{(1)}(n\cdot p, p^2, M_1) + \not{n}\,\frac{p^2}{n\cdot p}\,\Sigma_n^{(1)}(n\cdot p, p^2, M_1)\Big]\,e^2\,\Gamma_{ff_1V_1}^{\,2}\ . \tag{1}$$

The coupling factor $\Gamma_{ff_1V_1}$ is defined according to

$$\Gamma_{ff_1V_1} \quad = \quad V_{ff_1V_1} - \gamma_5\,A_{ff_1V_1}\ , \tag{2}$$

where $V_{ff_1V_1}$ and $A_{ff_1V_1}$ are the vector and axial-vector couplings of the fermion f to the exchanged gauge boson V_1. In our convention these coupling factors read

$$\Gamma_{ff\gamma} \quad = \quad -\,Q_f, \qquad \Gamma_{ffZ} = \frac{(1-\gamma_5)\,I_f^3 - 2\,Q_f\,\sin^2\theta_{\rm w}}{2\cos\theta_{\rm w}\,\sin\theta_{\rm w}}, \qquad \Gamma_{ff'W} = \frac{(1-\gamma_5)}{2\sqrt{2}\,\sin\theta_{\rm w}}\ . \tag{3}$$

Here I_f^3 is the third component of the weak isospin, $e\,Q_f$ is the electromagnetic charge, and $\theta_{\rm w}$ is the weak mixing angle.

[1]Note that similar simplifications can probably be obtained equally well by working in an axial gauge, see for instance Ref. [10] for massless particles.

The contribution to the external wave-function factor now amounts to multiplying the self-energy by $i/\not{p}$ on the side where it is attached to the rest of the scattering diagram and by the appropriate fermion source on the other side. Finally the square root should be taken of the external wave-function factor, i.e. the one-loop contribution should be multiplied by the usual factor 1/2. For an initial-state fermion, for example, one obtains[2]

$$\begin{aligned}\frac{1}{2}\,\frac{i}{\not{p}}\left[-i\,\Sigma^{(1)}(p,n,M_1)\right]u_f(p) \;&\approx\; \frac{e^2}{2}\,\Gamma^{\,2}_{ff_1V_1}\left[\Sigma^{(1)}_p(n\cdot p,m_f^2,M_1)+2\,\Sigma^{(1)}_n(n\cdot p,m_f^2,M_1)\right]u_f(p)\\ &\equiv\; \frac{1}{2}\,\Sigma^{(1)}_f(s,m_f^2,M_1)\,u_f(p)\;, \end{aligned}\tag{4}$$

where m_f is the mass of the external fermion and $\sqrt{s}=2\,p_0$ is the center-of-mass energy of the process $e^+e^-\to f\bar{f}$. Therefore the quantity of interest is actually $\Sigma^{(1)}_f$, which can be extracted from the full fermionic self-energy by means of the projection

$$\begin{aligned}\Sigma^{(1)}_f(s,m_f^2,M_1) \;&=\; \frac{i}{2\,p_0}\,\bar{u}_f(p)\left\{\frac{\partial}{\partial p_0}\left[-i\,\Sigma^{(1)}(p,n,M_1)\right]\right\}u_f(p)\\ &\approx\; -\,e^2\,\Gamma^{\,2}_{ff_1V_1}\int\frac{\mathrm{d}^4k_1}{(2\,\pi)^4}\,\frac{4\,p_\mu\,p_\nu}{[(p-k_1)^2-m_{f_1}^2+i\epsilon]^{\,2}}\,P^{\mu\nu}(k_1,n)\;, \end{aligned}\tag{5}$$

containing the gauge-boson propagator in the Coulomb gauge

$$P^{\mu\nu}(k_1,n) \;=\; \frac{-i}{k_1^2-M_1^2+i\epsilon}\left[g^{\mu\nu}+\frac{k_1^\mu k_1^\nu-k_1^0\,(k_1^\mu n^\nu+n^\mu k_1^\nu)}{\vec{k}_1^{\,2}}\right]\;. \tag{6}$$

Note that the loop-momentum k_1 has been neglected in the numerator of the fermion propagator, since only collinear-soft gauge-boson momenta will give rise to the Sudakov logarithms. The mass of the fermion inside the loop, m_{f_1}, is at best of the order of the Z-boson mass (for the top-quark). At the leading-logarithmic level it therefore only enters as an independent mass scale if the exchanged gauge boson is a photon (i.e. $m_{f_1}=m_f$), where it is needed for the regularization of collinear singularities. Therefore we restrict the arguments of $\Sigma^{(1)}_f$ to the energy and mass of the emitting fermion and the mass of the exchanged gauge boson, which are the three relevant scales for describing the Sudakov logarithms.

Having two canonical momenta at our disposal, i.e. p and n, we define the following Sudakov parametrisation of the gauge-boson loop-momentum k_1:

$$k_1=v_1\,q+u_1\,\bar{q}+k_{1_\perp}\;, \tag{7}$$

with

$$\begin{aligned} &p^\mu\equiv(E,\beta_fE,0,0)\;, &&\beta_f=\sqrt{1-m_f^2/E^2}\;, &&s=4\,E^2,\\ &q^\mu=(E,E,0,0)\;, &&\bar{q}^\mu=(E,-E,0,0)\;, &&k_{1_\perp}^\mu=(0,0,\vec{k}_{1_\perp})\;. \end{aligned}\tag{8}$$

[2] For an outgoing fermion one obtains $\frac{1}{2}\,\bar{u}_f(p)\,\bar{\Sigma}^{(1)}_f(s,m_f^2,M_1)$, where $\bar{\Sigma}^{(1)}_f$ can be derived from $\Sigma^{(1)}_f$ by reversing the sign in front of γ_5.

In terms of this parametrisation, the integration measure d^4k_1, the invariants $(p \cdot k_1)$ and k_1^2, and the gauge-boson energy k_1^0 read

$$\begin{aligned}
\mathrm{d}^4k_1 &= \pi\,\frac{s}{2}\,\mathrm{d}v_1\,\mathrm{d}u_1\,\mathrm{d}\vec{k}_{1_\perp}^{\,2}\ , \\
(p\cdot k_1) &= \frac{s}{4}\,[\,v_1\,(1-\beta_f)+u_1\,(1+\beta_f)\,] \approx \frac{s}{2}\Big(u_1+\frac{m_f^2}{s}\,v_1\Big)\ , \\
k_1^2 &= s\,v_1\,u_1-\vec{k}_{1_\perp}^{\,2} \qquad \text{and} \qquad k_1^0 = \frac{\sqrt{s}}{2}\,(v_1+u_1)\ .
\end{aligned} \tag{9}$$

The term containing the fermion mass m_f is needed for the exchange of photons only, regulating the collinear singularity at $u_1 = 0$. For the exchange of a massive gauge boson the mass M_1 will be the dominant collinear as well as infrared regulator.

The v_1-integration is restricted to the interval $0 \leq v_1 \leq 1$, as a result of the requirement of having poles in both hemispheres of the complex u_1-plane. The residue is then taken in the lower hemisphere in the pole of the gauge-boson propagator: $s\,v_1\,u_1^{\mathrm{res}} = \vec{k}_{1_\perp}^{\,2} + M_1^2 \equiv s\,v_1\,y_1$. Finally, $\vec{k}_{1_\perp}^{\,2}$ is substituted by y_1, with the condition $\vec{k}_{1_\perp}^{\,2} \geq 0$ translating into $v_1\,y_1 \geq M_1^2/s$. The one-loop Sudakov contribution to $\Sigma_f^{(1)}$ now reads

$$\begin{aligned}
\Sigma_f^{(1)}(s,m_f^2,M_1) &\approx -\frac{\alpha}{\pi}\,\Gamma_{ff_1V_1}^2\int_0^\infty \mathrm{d}y_1\int_0^1 \mathrm{d}v_1\,\frac{\Theta(v_1y_1-\frac{M_1^2}{s})}{(y_1+\frac{m_f^2}{s}v_1)\,(v_1+y_1)} \\
&\approx -\frac{\alpha}{\pi}\,\Gamma_{ff_1V_1}^2\int_0^1\frac{\mathrm{d}y_1}{y_1}\int_{y_1}^1\frac{\mathrm{d}z_1}{z_1}\,\mathcal{K}^{(1)}(s,m_f^2,M_1,y_1,z_1)\ ,
\end{aligned} \tag{10}$$

with the integration kernel $\mathcal{K}^{(1)}$ given by

$$\mathcal{K}^{(1)}(s,m_f^2,M_1,y_1,z_1) = \Theta\Big(y_1z_1-\frac{M_1^2}{s}\Big)\,\Theta\Big(y_1-\frac{m_f^2}{s}\,z_1\Big)\ . \tag{11}$$

Here we introduced the energy variable $z_1 = v_1 + y_1$ and made use of the fact that only collinear-soft gauge-boson momenta are responsible for the quadratic large-logarithmic effects: $y_1,\ z_1 \ll 1$. As a result, the gauge boson inside the loop is effectively on-shell and transversely polarized.

The exchanged gauge boson can either be a massless photon (γ) or one of the massive weak bosons (W or Z). The associated mass gap gives rise to distinctive differences in the two types of contributions (see Sect. 2.3). Bearing in mind that the SM is not parity conserving, we sum over the gauge-boson contributions and present the full one-loop Sudakov correction factor for right- and left-handed fermions/antifermions separately:

$$\delta_{f_R}^{(1)} = \delta_{\bar{f}_L}^{(1)} = \left(\frac{Y_f^R}{2\cos\theta_{\mathrm{w}}}\right)^2 \mathrm{L}(M,M) + Q_f^2\Big[\mathrm{L}_\gamma(\lambda,m_f)-\mathrm{L}(M,M)\Big]\ , \tag{12a}$$

$$\delta_{f_L}^{(1)} = \delta_{\bar{f}_R}^{(1)} = \left[\frac{C_F}{\sin^2\theta_{\mathrm{w}}}+\left(\frac{Y_f^L}{2\cos\theta_{\mathrm{w}}}\right)^2\right]\mathrm{L}(M,M) + Q_f^2\Big[\mathrm{L}_\gamma(\lambda,m_f)-\mathrm{L}(M,M)\Big]\ , \tag{12b}$$

with

$$\mathrm{L}(M_1, M_2) \;=\; -\frac{\alpha}{4\,\pi}\,\log\left(\frac{M_1^2}{s}\right)\log\left(\frac{M_2^2}{s}\right)\;, \tag{13}$$

$$\mathrm{L}_\gamma(\lambda, M_1) \;=\; -\frac{\alpha}{4\,\pi}\left[\log^2\left(\frac{\lambda^2}{s}\right)-\log^2\left(\frac{\lambda^2}{M_1^2}\right)\right]\;. \tag{14}$$

Note that these correction factors are the same for incoming as well as outgoing particles. In Eq. (12) $Y_f^{R,L}$ denotes the right- and left-handed hypercharge of the external fermion, which is connected to the third component of the weak isospin I_f^3 and the electromagnetic charge $e\,Q_f$ through the Gell-Mann – Nishijima relation $Q_f = I_f^3 + Y_f^{R,L}/2$. The coefficient $C_F = 3/4$ is the Casimir operator in the fundamental representation of $SU(2)$ and λ is the fictitious (infinitesimally small) mass of the photon needed for regularizing the infrared singularity at $z_1 = 0$. For the sake of calculating the leading Sudakov logarithms, the masses of the W and Z bosons can be represented by one generic mass scale M. Note that the terms proportional to Q_f^2 in Eq. (12) are the result of the mass gap between the photon and the weak bosons. Later on such mass-gap contributions will be the origin of non-exponentiating terms.

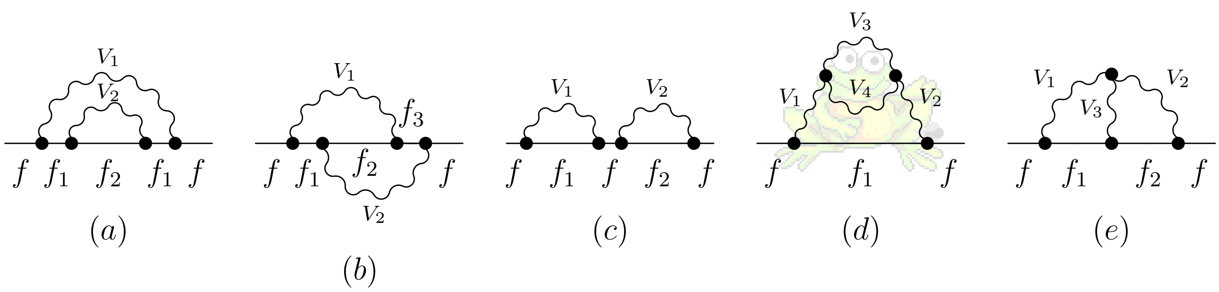
In the process $e^+e^- \to f\bar{f}$ the one-loop correction factors presented in Eq. (12) contribute in the following way to the polarized matrix element, bearing in mind that at high energies the helicity eigenstates are equivalent to the chiral eigenstates:

$$\mathcal{M}^{1-\text{loop, sudakov}}_{e_R^+e_L^-\to f_L\bar{f}_R} \;=\; \frac{1}{2}\left[\delta^{(1)}_{e_R^+}+\delta^{(1)}_{e_L^-}+\delta^{(1)}_{f_L}+\delta^{(1)}_{\bar{f}_R}\right]\mathcal{M}^{\text{born}}_{e_R^+e_L^-\to f_L\bar{f}_R}\;, \tag{15}$$

and similar expressions for the other possible helicity combinations.

2.2 The fermionic self-energy at two-loop level

At two-loop accuracy one has to take the following five generic sets of diagrams into account:



The fermions f_i are fixed by the exchanged gauge bosons V_i. Various cancellations are going to take place between all these diagrams. In unbroken theories like QED and QCD merely the so-called 'rainbow' diagrams of set (a) survive. The same holds if all gauge bosons of the theory would have a similar mass. The unique feature of the SM is that it is only partially broken, with the electromagnetic gauge group $U(1)_{\text{em}} \neq U(1)_Y$ remaining unbroken. As such three of the four gauge bosons will acquire a mass, whereas the photon remains massless and will interact with the charged massive gauge bosons ($W^\pm$). As a result, the 'rainbow' diagrams are not going to be the only contributions that survive the gauge cancellations.

The generic two-loop contribution of Sudakov logarithms to $\Sigma_f^{(2)}$ reads:

$$\Sigma_f^{(2)} \approx \left(-\frac{\alpha}{\pi}\right)^2 \Gamma_f^{(2)} \int_0^1 \frac{\mathrm{d}y_1}{y_1} \int_{y_1}^1 \frac{\mathrm{d}z_1}{z_1} \int_0^1 \frac{\mathrm{d}y_2}{y_2} \int_{y_2}^1 \frac{\mathrm{d}z_2}{z_2} \, \mathcal{K}^{(2)}(y_1, z_1, y_2, z_2) \, . \tag{16}$$

For the five different topologies the various products $\Gamma_f^{(2)} \times \mathcal{K}^{(2)}$ of coupling factors and integration kernels are given by

$$\begin{aligned}
\text{set (a):} \quad & \Big[\Gamma^2_{ff_1V_1}\, \mathcal{K}^{(1)}(s, m_f^2, M_1, y_1, z_1)\Big] \Big[\Gamma^2_{f_1f_2V_2}\, \mathcal{K}^{(1)}(s, m_f^2, M_2, y_2, z_2)\Big]\, \Theta(y_2 - y_1) \;, \\
\text{set (b):} \quad & -\Gamma_{ff_1V_1}\, \Gamma_{f_1f_2V_2}\, \Gamma_{f_2f_3V_1}\, \Gamma_{ff_3V_2}\, \mathcal{K}^{(1)}(s, m_f^2, M_1, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, M_2, y_2, z_2) \;, \\
\text{set (c):} \quad & \Big[\Gamma^2_{ff_1V_1}\, \mathcal{K}^{(1)}(s, m_f^2, M_1, y_1, z_1)\Big] \Big[\Gamma^2_{ff_2V_2}\, \mathcal{K}^{(1)}(s, m_f^2, M_2, y_2, z_2)\Big] \;, \\
\text{set (d):} \quad & -\Gamma_{ff_1V_1}\, \Gamma_{ff_1V_2}\, G_{134}\, G_{234} \Big[\mathcal{K}^{(1)}(s, m_f^2, M_3, y_2, z_2) + \mathcal{K}^{(1)}(s, m_f^2, M_4, y_2, z_2)\Big] \times \\
& \qquad \times\; \mathcal{K}^{(1)}(s, m_f^2, M_{12}, y_1, z_1)\, \Theta(y_2 - y_1)\, \Theta(z_1 - z_2) \;, \\
\text{set (e):} \quad & \frac{1}{4}\, \Gamma_{ff_1V_1}\, \Gamma_{f_1f_2V_3}\, \Gamma_{ff_2V_2}\, G_{132} \Bigg\{ \Big[\mathcal{K}^{(1)}(s, m_f^2, M_1, y_1, z_1) + \mathcal{K}^{(1)}(s, m_f^2, M_2, y_1, z_1)\Big] \times \\
& \qquad \times\; \mathcal{K}^{(1)}(s, m_f^2, M_3, y_2, z_2)\, \Theta(y_2 - y_1) \Big[1 + 3\, \Theta(z_1 - z_2)\Big] \\
& \qquad\qquad + \mathcal{K}^{(1)}(s, m_f^2, M_1, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, M_2, y_2, z_2) \times \\
& \qquad \times \Big[3 + \Theta(y_1 - y_2)\, \Theta(z_2 - z_1) + \Theta(y_2 - y_1)\, \Theta(z_1 - z_2)\Big] \Bigg\} \;,
\end{aligned} \tag{17}$$

with $M_{12} = \max(M_1, M_2)$. The totally antisymmetric coupling $e\, G_{ijl}$ is the triple gauge-boson coupling with all three gauge-boson lines (i, j, l) defined to be incoming at the interaction vertex. In our convention this coupling is fixed according to $G_{\gamma W^+W^-} = 1$ and $G_{ZW^+W^-} = -\cos\theta_{\mathrm{w}}/\sin\theta_{\mathrm{w}}$. Note that several of the integration kernels involve a specific ordering in the energy variables z_i [in set (d) and part of set (e)] and/or the angular variables y_i [in sets (a),(d) and part of set (e)]. The angular ordering in set (a), for instance, is caused by the fact that in the outer (k_1) loop-integral the incoming fermion momentum is p, yielding $(p - k_1)^2 \approx -s\, y_1$, whereas in the inner (k_2) loop-integral it is $p - k_1$, yielding $(p - k_1 - k_2)^2 \approx -s\,(y_1 + y_2)$. Therefore, only for $y_2 \gg y_1$ a large logarithm develops. The energy ordering occurs, for instance, when the soft gauge-boson momentum of the outer loop-integral is the incoming momentum of the inner loop-integral [see set (d)]. The Sudakov parametrisation for k_2 is based on k_1 in that case and the large logarithms only develop if $k_2^0 \ll k_1^0$. The multitude of terms contributing to the integration kernels of sets (d) and (e) originate from the various soft gauge-boson regimes that are possible in those diagrams. Note also the occurrence of the argument $\max(M_1, M_2)$ in the outer loop-integral of set (d), which only plays a role if one of the gauge bosons in the outer loop is a massless photon and the other a massive Z boson. Upon calculating the residues of the outer loop-integral in that case, we find that only the massive-particle residue contributes.

Note that certain diagrams look possible at first sight, but are in fact forbidden as a result of the charged current interactions of the W bosons. For instance, in set (b) it is not possible to exchange two W bosons without reversing the fermion-number flow (given by the direction of the Dirac propagator lines). Adding up all possible contributions, we find for the full two-loop Sudakov correction factor for right- and left-handed fermions/antifermions (see Sect. 2.3)

$$\delta_{f_R}^{(2)} = \delta_{\bar{f}_L}^{(2)} = \frac{1}{2}\left(\delta_{f_R}^{(1)}\right)^2 + Q_f^2\,\Delta_f \tag{18a}$$

$$\delta_{f_L}^{(2)} = \delta_{\bar{f}_R}^{(2)} = \frac{1}{2}\left(\delta_{f_L}^{(1)}\right)^2 + \left(Q_f^2 - \frac{|Q_f|}{2\,\sin^2\theta_{\rm w}}\right)\Delta_f\ , \tag{18b}$$

with

$$\Delta_f = \mathrm{L}(M,M)\left[\frac{4}{3}\,\mathrm{L}(M,m_f) - \mathrm{L}(M,M)\right] \tag{19}$$

and $\mathrm{L}(M_1,M_2)$ as defined in Eq. (13).

From Eqs. (18a) and (18b) we deduce our main statement, namely that the virtual electroweak two-loop Sudakov correction factor is not obtained by a mere exponentiation of the one-loop Sudakov correction factor. Based on the explicit two-loop calculation we find non-exponentiating terms, originating from the mass gap between the *massless* photon and the *massive* Z boson in the neutral sector of the SM. We have checked that these extra terms vanish in leading-logarithmic approximation if *all* gauge bosons have the same (or roughly the same) mass. From Eq. (19) it is clear that the non-exponentiating terms are genuine quadratic large-logarithmic effects. They will not vanish if the fermion mass is of the order of the masses of the weak bosons or if the energy is taken to infinity, since in those cases $\Delta_f \to \mathrm{L}^2(M,M)/3$. Therefore, as far as the virtual Sudakov logarithms are concerned, the SM will never completely behave like an unbroken theory, even not if the energy becomes arbitrarily large. This is a consequence of the fact that the photon is massless, i.e. $m_f/\lambda \gg \sqrt{s}/M$.

We also note that, in adding up all the contributions, we find that the 'rainbow' diagrams of set (a) yield the usual exponentiating terms plus an extra term similar to the one found in Ref. [6], originating from the charged-current interactions. Whereas in Ref. [6] this extra term was interpreted as the source of non-exponentiation, we observe that it in fact cancels against a specific term originating from the triple gauge-boson diagrams of set (e). Similar (gauge) cancellations take place between the 'crossed rainbow' diagrams of set (b), the reducible diagrams of set (c), and another part of the triple gauge-boson diagrams of set (e). Finally, most of the left-over terms of set (e) get cancelled by the contributions from the gauge-boson self-energy ('frog') diagrams of set (d). The term proportional to $|Q_f|$ in Eq. (18b) is the only surviving term of set (e), whereas the left–right symmetric term proportional to Q_f^2 originates from those diagrams of set (d) that involve neutral gauge bosons in the outer loop. The cancellation that usually takes place in unbroken gauge theories is upset by the fact that the on-shell poles for photons and Z bosons do *not* coincide, leading to different results for the on-shell residues. As can be seen from the integrals listed in Eq. (23) of Sect. 2.3, the energies of both the photon and the Z-boson are in the weak (soft-energy) domain, $z_1 \geq M/\sqrt{s}$, owing to the energy ordering. The observed difference is therefore caused entirely by the differences in the collinear domain induced by the mass gap. Based on this observation we conclude that the statements in Ref. [6]

concerning the factorization and exponentiation properties of the Sudakov logarithms in the ultrasoft energy regime, $\lambda/\sqrt{s} \leq z_1 \leq M/\sqrt{s}$, are not contradicted by our analysis.

Comparing with the other two studies, we can make the following remarks. First of all, a treatment of pure weak gauge-boson effects without reference to the photonic interactions breaks gauge-invariance, since the photon has an explicit $SU(2)$ component. This holds even if the photon is treated fully inclusively as in Ref. [5]. Such a separation would require a very careful definition, for instance in terms of the typical energy regimes that govern the Sudakov effects of pure electromagnetic origin (ultrasoft energies) and collective electroweak origin (soft energies). Second, in contrast to Ref. [7] we do not observe the exponentiation of the virtual one-loop Sudakov logarithms. Since both our methods differ substantially, it is hard to pinpoint where the precise conceptual differences are residing. We therefore restrict ourselves to pointing out two possible sources of differences. In the dispersive method of Ref. [7] it is assumed that QCD-like diagrammatic cancellations will take place, whereas we find that such cancellations can be upset by the fact that the on-shell poles for photons and Z bosons do not coincide. The latter might also have repercussions on the dispersive method itself, since in the diagrams of set (d) with $V_1 = \gamma$ and $V_2 = Z$, or vice versa, it is not possible that both gauge-bosons are effectively on-shell simultaneously.

2.3 Some useful integrals

In order to make our analysis more accessible, we present in this subsection the relevant loop-integrals, using the generic notation

$$I^{(i)} = \Big(-\frac{\alpha}{\pi}\Big)^i \int_0^1 \frac{\mathrm{d}y_1}{y_1} \int_{y_1}^1 \frac{\mathrm{d}z_1}{z_1} \cdots \int_0^1 \frac{\mathrm{d}y_i}{y_i} \int_{y_i}^1 \frac{\mathrm{d}z_i}{z_i}\, \mathcal{K}^{(i)}(y_1, z_1, \ldots, y_i, z_i) \; . \tag{20}$$

At one-loop level two types of kernels occur:

$$\begin{aligned} \mathcal{K}^{(1)}(s, m_f^2, M, y_1, z_1) \; &: \; I^{(1)} = \mathrm{L}(M,M) \; , \\ \mathcal{K}^{(1)}(s, m_f^2, \lambda, y_1, z_1) \; &: \; I^{(1)} = \mathrm{L}_\gamma(\lambda, m_f) \; . \end{aligned} \tag{21}$$

The functions $\mathrm{L}(M_1, M_2)$ and $\mathrm{L}_\gamma(\lambda, M_1)$ are the ones defined in Eqs. (13) and (14). At two-loop level seven new types of kernels occur. Four have angular ordering:

$$\begin{aligned} \mathcal{K}^{(1)}(s, m_f^2, M, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, M, y_2, z_2)\, \Theta(y_2 - y_1) \; &: \; I^{(2)} = \frac{1}{2}\, \mathrm{L}^2(M,M) \; , \\ \mathcal{K}^{(1)}(s, m_f^2, \lambda, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, \lambda, y_2, z_2)\, \Theta(y_2 - y_1) \; &: \; I^{(2)} = \frac{1}{2}\, \mathrm{L}_\gamma^2(\lambda, m_f) \; , \\ \mathcal{K}^{(1)}(s, m_f^2, M, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, \lambda, y_2, z_2)\, \Theta(y_2 - y_1) \; &: \; I^{(2)} = \frac{7}{12}\, \mathrm{L}^2(M,M) \; , \\ \mathcal{K}^{(1)}(s, m_f^2, \lambda, y_1, z_1)\, \mathcal{K}^{(1)}(s, m_f^2, M, y_2, z_2)\, \Theta(y_2 - y_1) \; &: \; I^{(2)} = \mathrm{L}(M,M)\, \mathrm{L}_\gamma(\lambda, m_f) \\ &\qquad - \frac{7}{12}\, \mathrm{L}^2(M,M) \; , \end{aligned} \tag{22}$$

and three have double ordering in both angle and energy:

$$
\begin{aligned}
\mathcal{K}^{(1)}(s,m_f^2,M,y_1,z_1)\,\mathcal{K}^{(1)}(s,m_f^2,M,y_2,z_2)\,\Theta(y_2-y_1)\,\Theta(z_1-z_2) \quad &: \quad I^{(2)} = \frac{1}{4}\,\mathrm{L}^2(M,M)\ , \\
\mathcal{K}^{(1)}(s,m_f^2,M,y_1,z_1)\,\mathcal{K}^{(1)}(s,m_f^2,\lambda,y_2,z_2)\,\Theta(y_2-y_1)\,\Theta(z_1-z_2) \quad &: \quad I^{(2)} = \frac{1}{3}\,\mathrm{L}^2(M,M)\ , \\
\mathcal{K}^{(1)}(s,m_f^2,\lambda,y_1,z_1)\,\mathcal{K}^{(1)}(s,m_f^2,M,y_2,z_2)\,\Theta(y_2-y_1)\,\Theta(z_1-z_2) \quad &: \quad I^{(2)} = \frac{2}{3}\,\mathrm{L}(M,M)\,\mathrm{L}(M,m_f) \\
&\qquad\qquad - \frac{1}{4}\,\mathrm{L}^2(M,M)\ . \qquad (23)
\end{aligned}
$$

Note that in the case of double ordering the collinear cut-off m_f^2 of the y_2 integral is in fact redundant.

3 Conclusions and Outlook

In order to settle the controversy in the literature concerning the resummation of virtual Sudakov logarithms in the process $e^+e^- \to f\bar{f}$, we have calculated these Sudakov logarithms at one- and two-loop level in the Coulomb gauge. In this special gauge all the relevant contributions, involving the exchange of collinear-soft gauge bosons, are contained in the self-energies of the external on-shell particles.

Our one-loop results are in agreement with the calculations in the literature. At two-loop level, however, we do not observe a mere exponentiation of the one-loop results, in contrast to claims in the literature. Moreover, our findings are in disagreement with another study that predicts non-exponentiation. The non-exponentiating terms in our two-loop result originate from the mass gap between the massless photon and the massive Z boson in the neutral sector of the SM. The cancellation that takes place in unbroken gauge theories, leading to exponentiation, is upset by the fact that the on-shell poles for photons and Z bosons do not coincide. As a result, not only the so-called 'rainbow' diagrams contribute at two-loop level. Also explicit contributions from diagrams with triple-gauge-boson interactions survive. We find that the corresponding non-exponentiating terms originate from the collinear domain and involve soft energies above the gauge-boson mass scale M.

From the explicit two-loop calculation we furthermore conclude that the non-exponentiating virtual Sudakov logarithms do not effectively vanish if the center-of-mass energy is taken to infinity. Therefore, as far as the virtual Sudakov logarithms are concerned, the SM will never completely behave like an unbroken theory at high energies. This is a consequence of the fact that the photon is strictly massless, being the gauge boson associated with the unbroken electromagnetic gauge group $U(1)_{\mathrm{em}}$.

For conclusive statements about the factorisation and exponentiation properties of the SM in the presence of experimental resolution cuts on energies and angles, the present study will have to be extended in order to account for real gauge-boson emission processes. It is conceivable that one can construct observables that are sufficiently inclusive with respect to final-state particles, energies, and angles, such that the SM effectively behaves like an unbroken theory.

The construction of such observables will rely heavily on the interplay (cancellations) between virtual and real effects. Since the Sudakov logarithms originate from the exchange of soft, effectively on-shell gauge bosons, many of the features derived for the virtual corrections will be intimately related to properties of the corresponding real-emission processes. The study of the virtual corrections presented in this letter therefore constitutes a substantial step towards a complete understanding of Sudakov effects in the SM.

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